ON THE PROPAGATION OF DISTURBANCES IN PLANE MAGNETOHYDRODYNAMIC FLOWS

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In [1] it was shown that in plane flow of an ideal gas with infinite electrical conductivity there are either two or four real characteristics, along which certain relations are satisfied. In computing flows on a linear theory it is necessary to know along which of these characteristics disturbances will die out, going away from the body toward infinity.

Below, the nature of the propagation of disturbances from breaks in streamlines is investigated. It is shown that shock waves may emerge from convex corners. Conversely, at concave corners the turning of the flow may be accomplished by waves of the Prandtl-Meyer type.

The analysis of the propagation of disturbances is applied to the investigation of the nature of the flow past bodies with a small angle between the directions of the magnetic field and the direction of the approaching flow.

1. Outgoing and incoming shock waves. Let us investigate the flow changes in a weak shock wave for the case where the vectors of the velocity $\mathbf{V}_{0||}$ and magnetic field $\mathbf{H}_{||}$ are parallel (Fig. 1). If we denote by $\sigma_{0||}$ the angle between the characteristics and the flow direction ahead of the wave, and by $\sigma_{1||}$ the corresponding angle behind the wave, then, as shown in [2], $\sigma_{1||} > \sigma_{c||} > \sigma_{0||}$, that is, the shock divides the angle between the characteristics. It is evident that the portion of the wave DO is incoming, in relation to the streamline AOB, since the characteristics which run into the wave come from below, and the disturbances which propagate along them, are not determined by the streamline AOB. Conversely, the portion of the wave OE depends on the streamline AOB and is outgoing.

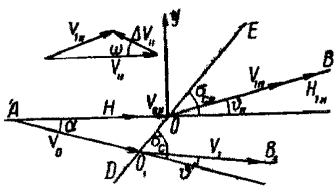

Fig. 1.

Now let the field vector $\mathbf{H} = \mathbf{H}_{||}$ make an angle α with the velocity vector $\mathbf{V}_0$ ahead of the wave (Fig. 1). To go to this case from the one considered above, it is sufficient to add to the flow a velocity $\mathbf{V}$, parallel to the wave, such that $\mathbf{V}_0 + \mathbf{V} = \mathbf{V}_{0||}$. Evidently, $\mathbf{V}_1 - \mathbf{V}_0 = \mathbf{V}_{1||}$ y $\mathbf{V}_{0||} = \Delta\mathbf{V}_{||}$. In [2] it is shown that, to an accuracy $\theta_{||}{}^2$,

$$\frac{\Delta V_{\parallel y}}{\Delta V_{\parallel x}} = -\operatorname{tg}\omega = (1 - M_{0\parallel}^2)\operatorname{tg}\sigma_{0\parallel} \qquad (1.1)$$

It is evident that for $\alpha > \omega$ the angle θ is negative, i.e. the convex side of the streamline AO_1B_1 faces upward. For $\alpha = \omega$ the flow does not turn at the shock wave.

Let us investigate how the character of the approach of the characteristics to the shock wave changes with a change of the angle α. Denote by σ_c the angle of inclination of a characteristic of the oncoming flow to the velocity vector $\mathbf{V}_0$. In the case of flow with parallel velocity and field vectors, the corresponding characteristic has inclination $\sigma_{||}$ and the corresponding velocity is $V_{||}$, which are determined by the relations [1]

$$V_0 = \frac{V_{\parallel}}{\cos\alpha}\frac{\operatorname{tg}\sigma_{\parallel}}{\operatorname{tg}\sigma_{\parallel} + \operatorname{tg}\alpha}, \qquad \operatorname{tg}\sigma_0 = \frac{\operatorname{tg}\sigma_{\parallel} + \operatorname{tg}\alpha}{1 - \operatorname{tg}\sigma_{\parallel}\operatorname{tg}\alpha} \qquad (1.2)$$

$$\operatorname{tg}\sigma_{\parallel} = \pm\sqrt{\frac{M_{\parallel}^2 - N^2(1 - M_{\parallel}^2)}{(1 - M_{\parallel}^2)(N^2 - M_{\parallel}^2)}} \qquad \left(N^2 = \frac{H_0^2}{2\pi\varkappa p_0}\right) \qquad (1.3)$$

where $M_{||}$ is the Mach number corresponding to $V_{||}$, and p_0 is the pressure in the oncoming flow. On the other hand,

$$V_0 = \frac{V_{0\parallel}}{\cos\alpha}\frac{\operatorname{tg}\sigma_{c\parallel}}{\operatorname{tg}\sigma_{c\parallel} + \operatorname{tg}\alpha}, \qquad \operatorname{tg}\sigma_{0\parallel} = \pm\sqrt{\frac{M_{0\parallel}^2 - N^2(1 - M_{0\parallel}^2)}{(1 - M_{0\parallel}^2)(N^2 - M_{0\parallel}^2)}} \qquad (1.4)$$

For small θ, evidently, $\sigma_{c||} = \sigma_{0||} + \delta$, $\sigma_{||} = \sigma_{0||} + \tau$, and $M_{||} = M_{0||} + \epsilon$, where δ, τ and ϵ are quantities of order θ. Putting these expressions in

(1.2) and (1.4), dropping terms of order θ^2, and eliminating ϵ with the help of (1.3), we obtain

$$\frac{\tau}{\delta} = \left\{1 + \frac{\mathrm{tg}^2\,\sigma_{0\parallel}\,(\mathrm{tg}\,\sigma_{0\parallel} + \mathrm{tg}\,\alpha)\,(M_{0\parallel}{}^2 - 1)^2\,(M_{0\parallel}{}^2 - N^2)^2}{M_{0\parallel}{}^4\,\mathrm{tg}\,\alpha\,[(N^2 - M_{0\parallel}{}^2) + N^2\,(1 - M_{0\parallel}{}^2)]}\right\}^{-1} \tag{1.5}$$

For $\tau/\delta < 1$, the characteristics going out from the streamline AO_1B_1 run into the shock above the streamline, and the wave going out from the body goes upwards; for $\tau/\delta > 1$, conversely, the characteristics from the streamline AO_1B_1 run into the shock below that line, and the wave goes out from the body downward.

Let $\theta_{\parallel} > 0$ and $M_{0\parallel} < 1$. In this case $\mathrm{tg}\,\sigma_{0\parallel} < 0$ [2]. For $a = 0$, evidently, $\tau = 0$. The characteristics from the streamline AOB approach the wave from below. With increasing a, the characteristics continue to approach from below, but the angle between them and the wave increases until, at some angle $a = a_*$, they become perpendicular to the wave. For still larger angles the characteristics approach the wave from above. The critical value $a = a_*$ corresponds to $\tau/\delta \to -\infty$, i.e.

$$\mathrm{tg}\,\alpha_* = -\,\mathrm{tg}\,\sigma_{0\parallel}(1 - M_{0\parallel}{}^2)\,\frac{(N^2 - M_{0\parallel}{}^2)\,[M_{0\parallel}{}^2 - N^2\,(1 - M_{0\parallel}{}^2)]}{(N^2 - M_{0\parallel}{}^2)\,[M_{0\parallel}{}^2 - N^2\,(1 - M_{0\parallel}{}^2)] + N^4 M_{0\parallel}{}^2\,(1 - M_{0\parallel}{}^2)} \tag{1.6}$$

Equating this expression to (1.1) we see that $a_* < \omega$, i.e. the characteristics "reverse themselves" for smaller values of a than does the streamline. Thus, for $a_* < a < \omega$, shock waves go out from a convex corner. For $-\,\mathrm{tg}\,\sigma_{0\parallel} > \mathrm{tg}\,a > \mathrm{tg}\,\omega$ the wave goes out from a concave corner, corresponding to $\theta < 0$. For $\mathrm{tg}\,a > -\,\mathrm{tg}\,\sigma_0$, it is evident that $\tau/\delta < 1$. This situation obtains also for $M_{0\parallel} > 1$.

An analogous analysis shows that for expansion waves of the Prandtl-Meyer type the points $a = a_*$ and $a = \omega$ also appear as singular points (for $\theta_{\parallel} < 0$). At $a = a_*$ the characteristic fan changes its direction to the other side, at $a = \omega$ the deflection angle of the velocity vector changes sign. In the interval $a_* < a < \omega$ the turning of the flow in a concave corner is of a Prandtl-Meyer type.

2. Flow at small values of a. In [1] it was shown that Equations (1.2) give a parametric (parameter $M_{0\parallel}$) solution of the fourth order equation which determines the inclination of the characteristics. Assigning a and $M_{0\parallel}$, we find M_0 and $\mathrm{tg}\,\sigma_0$.

In accordance with the classification in [1], regimes in which there are four real characteristics are called hyperbolic, and regimes with two real characteristics elliptic-hyperbolic. Knowing the relations between double arrow indicates the character of the approach of the characteristics to the wave, and, it follows, its direction. In the case considered the wave goes out from a corner at the point 0. Adding a velocity parallel to the wave we find (Fig. 2) that the velocity vector behind the wave, for $\alpha \neq 0$, must lie in the Quadrant $1'$. From (1.5) it follows that $\tau/\delta > 1$. Therefore, the characteristics approach the wave like the arrows A'. It follows that the wave goes downstream from a concave streamline on the upper surface of the body.

$$\text{(A2)} \qquad -1 < M_{\parallel} < 0, \qquad 0 > \operatorname{tg}\sigma_{\parallel} > -\operatorname{tg}\alpha, \qquad \operatorname{tg}\alpha > \operatorname{tg}\sigma > 0$$

Since here $|M_{\parallel}| < 1$, then in accordance with (1.1) the velocity vector after the wave for $\alpha = 0$ will appear on the line OC (Fig. 2) and the direction of the wave is defined by the arrows B. In the corresponding flow for $\alpha \neq 0$ the velocity vector behind the wave will lie on the

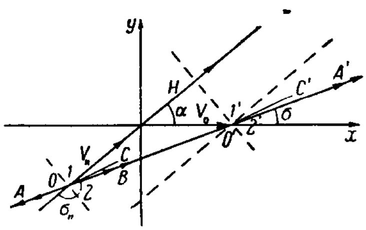

Fig. 2.

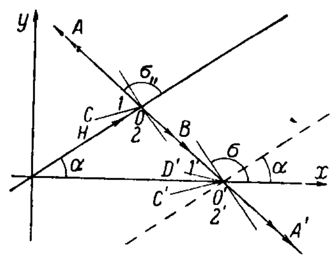

Fig. 3.

line $O'C'$, and the direction of the wave is given by arrows A', since here $\tau/\delta < 1$. Thus this wave has the same character as the preceding one.

$$\text{(A3)} \quad M_{\parallel} < 1, \quad \operatorname{tg}\sigma_{\parallel} = -k\operatorname{tg}\alpha, \quad \operatorname{tg}\sigma = (1-k)\operatorname{tg}\alpha, \quad 1 \leqslant k \leqslant 1 + N^2$$

From (1.2), (1.3) and (1.1) we easily find that $0 < \operatorname{tg}\omega < \operatorname{tg}\alpha$ for $k < 1 + N^2$ and $\operatorname{tg}\omega > \operatorname{tg}\alpha$ for $k > 1 + N^2$. It follows that the velocity vector after the wave (Fig. 3) must lie on the line $O'C'$, and the wave goes out, from a downward bend at O', in the direction of arrow A', since for $\alpha = 0$ it was in the direction of A, and $\tau/\delta > 1$.

$$\text{(A4)} \quad M_{\parallel} > 1, \quad \operatorname{tg}\sigma_{\parallel} < -\operatorname{tg}\alpha, \quad \operatorname{tg}\sigma < 0$$

The velocity diagram is given in Fig. 4. Since $M_{\parallel} > 1$, the end of the velocity vector will appear Quadrant $2'$. In accordance with (1.5) we have $\tau/\delta < 1$, and, it follows, the wave is again in the direction of arrow A', since in the corresponding flow with $\alpha = 0$ the wave is inclined in the direction of B.

flow parameters along characteristics [1], it is possible to construct, speaking in a general sense, the full solution in hyperbolic flows or to separate the hyperbolic part of the solution in elliptic-hyperbolic flows [1].

This is most simply done within the framework of linearized theory. But even to construct flows on a linear theory, it is necessary to clarify the possible shock wave configuration, since to calculate the flow it is necessary to know along which characteristics disturbances die out toward infinity. In the general case, disturbances can propagate upstream as well as downstream; therefore, to answer the above question, it is essential to clarify which of the characteristics do not actually go to infinity but run into the shock wave. In going out to infinity along these characteristics in the linearized approximation, disturbances need not die out.

Characteristics are shock waves of vanishing strength; therefore, there may exist as many shock waves as characteristics. However, some of them may be incoming waves, not dependent on the body. Evidently, such waves should not exist in the undisturbed flow at infinity.

Below, using the example of a thin airfoil section and with the help of the results in Article 1, the character of the flow for $a < 1$ will be investigated. For small a all the flow types exist, and at the same time the analysis is substantially simplified.

3. Hyperbolic flows. Hyperbolic flow occurs for $M > M_3 = [1 - 1/2(1 - N^2)\,\mathrm{tg}^2 a]\sec a$. Let us examine several regimes separately.*

(A) $$M > \sqrt{1+N^2}\csc\alpha$$

Here there are four waves [1]:

(A1) $$M_{\parallel} < -1, \quad 0 > \mathrm{tg}\,\sigma_{\parallel} > -\mathrm{tg}\,\alpha, \quad \mathrm{tg}\,\alpha > \mathrm{tg}\,\sigma > 0$$

On Figure 2 is shown a velocity diagram, corresponding to these parameters. In accordance with (1.1), the end of the velocity vector behind a wave with inclination $\sigma_{\parallel} > 1/2\,\pi$ will appear in the Quadrant 1. The

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* In the following it is not essential to distinguish between σ_c and σ_0; $\sigma_{c\parallel}$, $\sigma_{0\parallel}$ and $\sigma_{\parallel}$; M_0 and $M_{\parallel}$. Below, for each of these groups we shall use σ, $\sigma_{\parallel}$ and $M_{\parallel}$, respectively. Instead of M_0 we shall write M for simplicity.

Thus, the last two waves can go downstream from a concave stream-line on the lower surface. The general character of the waves on the airfoil section in the case considered is shown in Fig. 4a.

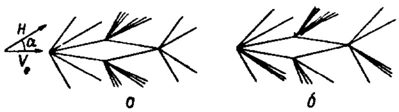

Fig. 4.

(B) $$\sqrt{1+N^2}/(N\cos\alpha) < M < \sqrt{1+N^2}\csc\alpha$$

Here there are again four waves [1]

(B1) $$-1 < M_{\parallel} < 0, \quad 0 > \operatorname{tg}\sigma_{\parallel} > -\operatorname{tg}\alpha, \qquad \operatorname{tg}\alpha > \operatorname{tg}\sigma > 0$$

wave of type (A2)

(B2) $$M_{\parallel} < 1, \quad \operatorname{tg}\sigma_{\parallel} = -k\operatorname{tg}\alpha, \quad \operatorname{tg}\sigma = (1-k)\operatorname{tg}\alpha, \ 1 \leqslant k < 1 + N^2$$

wave of type (A3)

(B3) $$M_{\parallel} > 1, \qquad \operatorname{tg}\sigma_{\parallel} < -\operatorname{tg}\alpha, \qquad \operatorname{tg}\sigma < 0$$

wave of type (A4)

(B4) $$M_{\parallel} > 1, \qquad \operatorname{tg}\sigma_{\parallel} > 0, \quad \operatorname{tg}\sigma > \operatorname{tg}\alpha$$

For the case (B4) the velocity diagram is shown on Fig. 5. It is easy to see that the velocity vector after the wave must lie in Quadrant 1′.

Furthermore, it may be seen from (1.1) and (1.3) that $\operatorname{tg}\omega \to \infty$ when $M_{\parallel} \to \infty$. Let us find the value of $M_{\parallel}$ for which $\omega = \alpha$. We have

$$\operatorname{tg}\alpha = \sqrt{\frac{[M_{\parallel}^2(1+N^2) - N^2](M_{\parallel}^2 - 1)}{M_{\parallel}^2 - N^2}}$$

It is evident that for $M_{\parallel} > 1$ this equation can be fulfilled if $M_{\parallel} = 1 + O(\alpha^2)$. Dropping terms of higher order, we obtain

$$M_{\parallel} = 1 + {}^1\!/_2(1-N^2)\operatorname{tg}^2\alpha, \quad \operatorname{tg}\sigma_{\parallel} = [(1-N^2)\operatorname{tg}\alpha]^{-1}$$
$$M = [1 - {}^1\!/_2(1-N^2)\operatorname{tg}^2\alpha]\sec\alpha$$

It is easy to show that these values correspond to the point $M = M_3$, where the curve $M = f(M_{\parallel})$ has a minimum [1]. It follows that in the range of values of M being considered we have $\omega > \alpha$, and the end of the velocity vector behind the wave lies on the line $O'C'$ (Fig. 5).

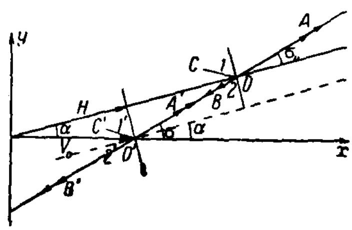

Fig. 5.

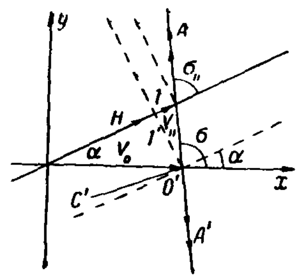

Fig. 6.

Furthermore, it is easy to show that along the curve in question $r/\delta < 1$. Therefore, the wave has the direction of the arrow A'. Thus, the general picture of the waves on the airfoil section (Fig. 4a) will be the same as in case (A).

(C) $$\sec\alpha \leqslant M \leqslant \sqrt{1+N^2}/N\cos\alpha$$

In this case there are three waves (C1), (C3), (C4) which are the same as waves (B1), (B3), and (B4). The wave (C2) differs from (B2) or (A3) only in that here $k > 1 + N^2$, and therefore, in accordance with what was said above (case (A3)), $\operatorname{tg}\omega > \operatorname{tg}\alpha$. It follows that the end of the velocity vector (Fig. 3) behind the wave lies on the line $O'D'$, i.e. at the wave the turn is through a positive angle θ. Thus, this wave can go downstream from convex corners on the lower surface of the airfoil section, and the general flow picture will be that depicted in Fig. 4b, where $|\operatorname{tg}\sigma_3| > \operatorname{tg}|\sigma_2|$.

(D) $$[1 - {}^1/_2(1-N^2)\operatorname{tg}^2\alpha]\sec\alpha < M < \sec\alpha$$

Waves (D1), (D2) and (D4) are identical with (C1), (C2) and (C4), respectively.

(D3) $$M_{\parallel} > 1, \quad \operatorname{tg}\sigma_{\parallel} > \operatorname{ctg}\alpha, \quad \operatorname{tg}\sigma < -\operatorname{ctg}\alpha$$

(в точке $M = M_3$ имеем $\operatorname{tg}\sigma_{\parallel} = \operatorname{ctg}\alpha$)

The corresponding velocity diagram is given in Fig. 6.

Above (case (B4)), it was shown that on the curve $M = f(M_{\parallel})$, we always have $\omega > \alpha$ and $r/\delta < 1$ to the right of the point $M = M_3$. It can be shown that to the left of that point, $\omega < \alpha$ and $r/\delta > 1$. Consequently, the end of the velocity vector behind the wave lies on $O'C'$ (Fig. 6), and the wave goes from a concave corner in the direction of arrow A', the flow picture being again as in Fig. 4b.

4. Elliptic-hyperbolic flow. These flows occur [1] for $M_3 > M > M_2$, regime (E), and for $0 < M < M_1$, regime (F). In each of these regimes there are two characteristics*.

In [1] it was shown that in this regime it is possible to divide the flow into a hyperbolic portion which does not die out at infinity and an elliptic portion which dies out. The non-decaying disturbances propagate along characteristics, running into shock waves.

In regime (E) there may exist waves of type (C1) and (C2). The flow picture is shown in Fig. 7a.

In regime (F) there are also two waves

(F1) (wave of type (B1))

(F2) $1 > M_{\parallel} > 0, \quad \operatorname{tg}\sigma_{\parallel} > 0, \quad \operatorname{tg}\sigma > \operatorname{tg}\alpha$

From the velocity diagram (Fig. 5) it may be seen that the end of the velocity vector behind the wave must lie in Quadrant 2', and, since according to (1.5) we have $r/\delta < 1$, the wave is in the direction of arrow B.

The general nature of the flow in this case is shown in Fig. 7b. Here also, an elliptic flow (dying out at infinity) is to be superimposed on the discontinuous hyperbolic part of the flow.

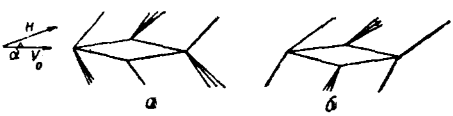

Fig. 7.

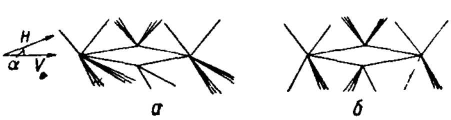

Fig. 8.

5. Quasi-hyperbolic flow. These flows occur [1] for $M_1 < M < M_2$. Here again there are four real characteristics. The curve $M = f(M_{\parallel})$ for $\operatorname{tg}\sigma_{\parallel} < 0$ has a maximum at the point M_2 and a minimum at the point M_1. For small α, the values of $M_{\parallel}$ corresponding to the points M_1 and M_2 lie in the neighborhood of $M_{\parallel} = N/\sqrt{1 + N^2}$ and $M_{\parallel} = N$, respectively.

Let $M_{\parallel}^2 = N^2 - k^2 \operatorname{tg}^2\alpha$. For $k = 0$, evidently, $M = N \sec\alpha$. For $k > 0$,

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* The values M_1 and M_2 are given in Article 5.

$$\operatorname{tg}\sigma_{\parallel} = \frac{-N^2}{k(1-N^2)\operatorname{tg}\alpha}, \qquad \operatorname{tg}\sigma = \frac{-N^2}{[N^2+(1-N^2)k]\operatorname{tg}\alpha}$$

$$M = \frac{N}{\cos\alpha}\left\{1 - \operatorname{tg}^2\alpha\left[\frac{k^2}{2N^2} - \frac{k(1-N^2)}{N^2}\right]\right\} \tag{5.1}$$

It is easy to prove that the point M_2 corresponds to $k = 1 - N^2$. For $k = 2(1 - N^2$ we again have $M = N \sec \alpha$.

Let us investigate the nature of the flow in the neighborhood of the point M_2

(G) $\quad N\sec\alpha < M < N\sec\alpha\,[1 + {}^1/_2(1-N^2)N^{-2}\operatorname{tg}^2\alpha]$

It is obvious that the waves (G1) and (G2) will be of type (C1) and (C2).

(G3) $\quad M_{\parallel} = N(1 - {}^1/_2 k^2N^{-2}\operatorname{tg}^2\alpha)$

for which $\operatorname{tg}\sigma_{\parallel}$ and $\operatorname{tg}\sigma$ are determined by (5.1) for $1 - N^2 < k < 2(1 - N^2)$.

The velocity diagram is given in Fig. 3. For sufficiently small α along the portion of the curve $M = f(M_{\parallel})$ (with $\operatorname{tg}\sigma_{\parallel} < 0$) which lies in the quasi-hyperbolic regime, $\omega > \alpha$. Substituting (5.1) in (1.5) it is easy to prove that $r/\delta < 1$ to the left of the point M_2. It follows that the wave under consideration is inclined in the direction of arrow A, and can go upstream from concave corners on the upper surface of the airfoil section.

The wave (G4) differs from the preceding ones only in that here $0 < k < (1 - N^2)$ and $r/\delta > 1$. Therefore, in accordance with Fig. 3, the wave must be inclined in the direction of the arrow A' and can go downstream from convex corners on the lower surface of the profile. Thus, the flow picture in this case will be as on Fig. 8a.

Let us now investigate the neighborhood of the point M_1. Let $M_{\parallel}^2 = N^2(1 + N^2)^{-1} + \epsilon^2$ and $\operatorname{tg}\sigma_{\parallel} = -k\operatorname{tg}\alpha$. Then, from (1.3) we have

$$M = \frac{N}{\cos\alpha}\,\frac{2k(1+N^2)^2 + k^3N^2\operatorname{tg}^2\alpha}{2(1+N^2)^{3/2}(k-1)}, \qquad \varepsilon = kN^2(1+N^2)^{-3/2}\operatorname{tg}\alpha \tag{5.2}$$

At the point M_1

$$k = (B\operatorname{tg}\alpha)^{-2/3} \qquad \left(B = \frac{N}{(1+N^2)}\right)$$

Then

$$M_1 = N(\cos\alpha\sqrt{1+N^2})^{-1}[1 + {}^1/_2(B\operatorname{tg}\alpha)^{2/3}][1-(B\operatorname{tg}\alpha)^{2/3}]^{-1}$$

$$\text{(H)} \qquad M_1 < M < N\sec\alpha$$

The waves (H1) and (H2) are identical with waves (C1) and (C2). It is easy to prove that to the right of point M_1, we must have $r/\delta < 1$, and the wave (H3) has the same character as (C3).

$$\text{(H4)} \qquad M_{\parallel} < 1, \qquad \operatorname{tg}\sigma_{\parallel} > 0, \qquad \operatorname{tg}\sigma > 0$$

The velocity diagram here corresponds to Fig. 5. Since here $r/\delta < 1$, and the end of the velocity vector behind the wave must lie in Quadrant 2′, therefore the wave is inclined in the direction of arrow B', and the flow picture has the form shown in Fig. 8b.

We may note that the flow picture depicted in Figs. 4, 7 and 8 is constructed with the assumption that at any corner the turning of the flow by every wave is in the same direction. Generally speaking, this is not always the case, an example of this being given in [1]. It may happen, for instance, that in the first shock wave the flow turns through an angle larger than that of the body, then following it there may be an expansion wave instead of a second shock wave. This question has to be decided separately for each particular case. In the present investigation what is important is that along characteristics which coincide with shock waves, disturbances need not die out.

We also note that if the contour does not have a corner then a detached shock wave will be formed at a certain distance from the airfoil section.

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